PROPOSAL OF A GAIN-SCHEDULING METHOD TO CONTROL THE TWO-WHEELED ROBOT FOR WALKING ASSISTANCE

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Abstract: *A completed hardware and performance test results of a two-wheeled robot to support the elder in walking and standing are presented in this paper. The two-wheeled robot is based on the inverted pendulum which is linearized by utilizing the lie algebra method. The hardware of the robot is designed accurately and safe for users. Especially, the gain-scheduling method for a nonlinear system is applied to the two-wheeled robot to enhance its performance. The experimental results verified that the proposed gain-scheduling method improves greatly the performance of the two-wheeled robot that the time reponse is instantaneous as well as the stability is very good. The effectiveness of the gain-scheduling method is also verified much better than only used LQR (Linear Quadratic Regulator LQR) method. Thus, the two-wheeled robot is really effective to assist the elder or the disable in moving..*

Keywords: *Inverted pendulum, Robotic cane, Assist device, Gain-scheduling, Nonlinear disturbance observer.*

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I. INTRODUCTION

Following the rapid growth of the global aging population, medical and health needs are increasingly urgent. Especially, the need for assistive devices for the elderly or the disable in rehabilitation centers in developed countries.

Assistive traditional devices are in rudimentary form, need to be supported by additional people in operation like wheel-chairs. Besides, these traditional devices do not have many benefits and are not comfortable for users.

A research of the wheel-chair robot using omnidirectional wheels to support the user's movement is given in [1]. This robot has the ability to flexibly move in different directions. But the wheelchair design makes it difficult to move on different terrains, and the space of movement is also limited because of its big size.

Control techniques as suggested by the authors in [2] significantly improved the operation performance of the robot to assist patients at rehabilitation centers to restore mobility, but the robot has many limitations such as cumbersome and complex structure to move.

In order to enhance the versatility of assistive devices in rehabilitation centers, the authors in [3] have presented a new robot model in the form of three wheels, but it is also quite cumbersome and can not move on rough terrains with a low friction.

In addition, a miniaturized form of the 3-wheels robot model was further studied in [4]. Remarkable results were achieved such as the applicability of the position controller for the robot control problem. The robot adopts multi-directional wheels to help support the user more flexibly. However, the robot can not support in outsides environment.

Another robot was designed to help the user maintain an upright state by lifting the body by two extra legs attached to the human body, which is quite stable and useful, but attaching the device to the user becomes cumbersome and less flexible in practice. This robot is also shown similarly to devices presented in [10-11]. These devices are attached to the user's leg joints leading to uncomfortable feelings. These models may be only applied at rehabilitation centers.

The omnidirectional 3-wheel robot model in [13] was developed to enhance interaction between the user and the robot as well as make more assistance for the user from the robot with positive feedbacks. However, reducing the robot demension is still challenging, and the operation of the robot in real environment is limited.

The motion balance of the nonlinear system has been researched and developed in [5], and applied quite well and effectively on the inverted pendulum model. Next, the development of this research in [6], based on the nonlinear disturbance monitoring system, makes the motion control of the robot become more flexible, depending on the influence (force) of the user. The results in [5] and [6] were developed in [7-8] with experiments on different inverted pendulum models such as 1-wheeled stick robot [7] and 2-wheeled stick robot [8]. However, different impacts from users have not been studied and tested on these robots.

The application of the nonlinear disturbance observer to the inverted pendulum model was analyzed and applied in [12] with reliable results, but the user's impacts in motion at different angle positions have not been considered. Thus, this proposed model is still not suitable in maintianing balance for the user when moving.

In order to enhance the response of the control system in different operation environments with a high accuracy, a gain scheduling method was proposed in [14-15] for a nonlinear control system. Thanks to this method, the nonlinear control system works more flexibly and smoothly. The gain scheduling method has been studied and applied on the inverted pendulum model in [16] and good simulation results were achieved in different deviation angles, however, the verification of this controller has not been proven on a real model.

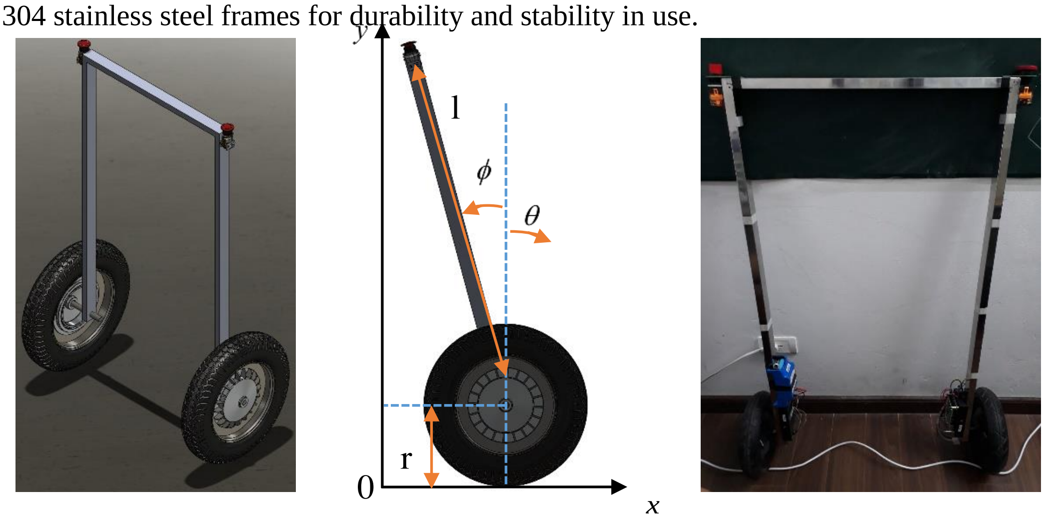
Control methods such as PID, LQR and MPC for the inverted pendulum model of the two-wheeled robot have been studied and tested in [17]. The robot works well with the robot's deviation around the equilibrium point (approximately zero point). However, in order to apply the robot in real life to support the user, the deviation of the robot has to be much bigger than rezo, not only around the equilibrium point. In this paper, this disadvantage will be solved and tested on a real robot.

From the above studies, this paper will propose a new gain-scheduling method for a nonlinear system, and then, it is applied on a two-wheeled robot based on an inverted pendulum model to improve the responsibility of the robot depending on the behavior of the user without falling down. The performance of the controller of the two-wheeled walker is verified by experimental results which show that the two-wheeled robot is really useful to support the elder or the disable in walking or standing.

The paper is organized as follows: Section 2 presents designed hardware model in detail. The mathematical models of the two-wheeled robot are alalyzed, and the proposed gain-scheduling method is also presented in Section 3. The performance of the proposed gain-scheduling method applied to the two-wheeled robot verified by experiments and measured results on real users is shown in Section 4.

II. HARDWARE DESIGN OF THE TWO-WHEELED ROBOT

In this Section, the hardware of the two-wheeled robot is revealed. First, the robot is drawn by the software Solidworks as shown in Fig. 1 (a). The robot consists of a center controller, brushless motor controllers, a lithium battery, a charge controller circuit, a communication circuit, and an accelerometer sensor. The robot is based on the mathematical model the inverted pendulum in Figure 1(b). The complete hardware design for the two-wheeled robot is shown in Figure 1 (c) after the basic parts of the two-wheeled robot are designed, and the mechanical processing is made by 304 stainless steel frames for durability and stability in use.



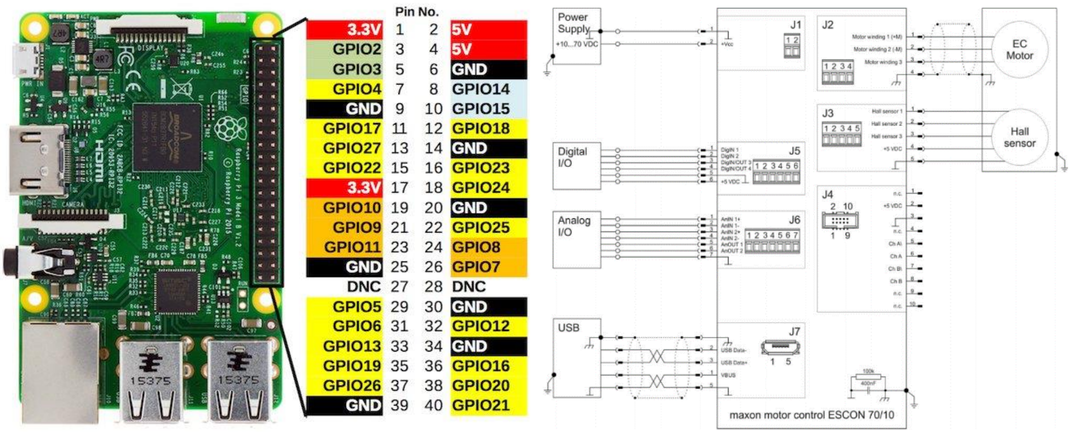
(a) Designed of two-wheeled robot in Solidworks

(b) Analysis of inverted pendulum model

(c) Actual two-wheeled robot after manufacturing

Figure 1. *The two-wheeled robot.*

First, the central controller handles control algorithms, collects information from sensors and gives control commands to the motion motor. It is an embedded computer Raspberry Pi 3 Model B + with 2GB RAM and 16GB memory as shown in Figure 2(a).

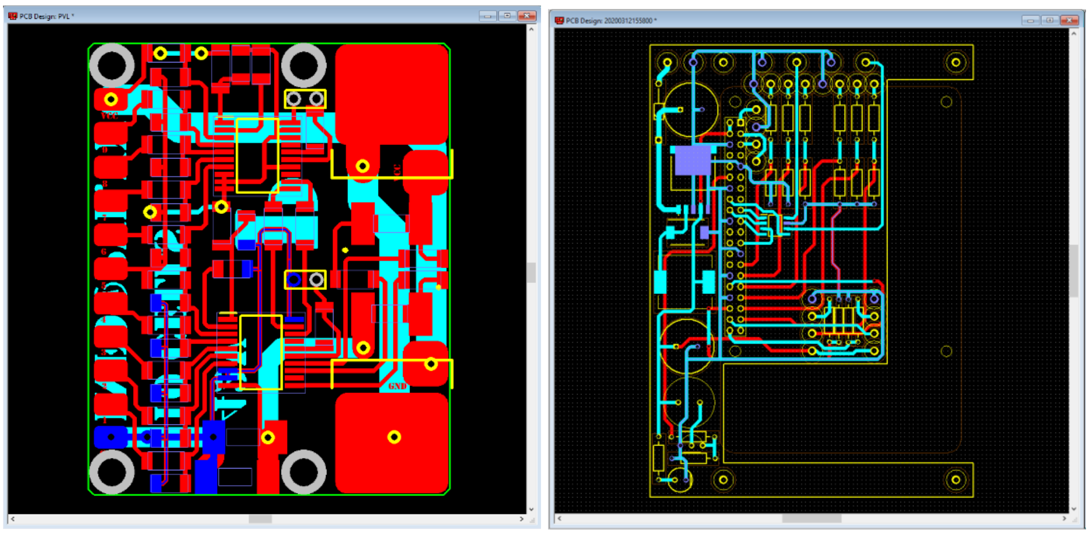


(a) *(b)*

Figure 2. *Center controller (a) and block diagram of motor driver (b).*

The second component is ESCON-70/10 brushless motor controllers which are responsible for receiving analog control signals from the central controller to control the position of the wheels through the current ranging from -10A to 10A with the input voltage between 0VDC and 70VDC as shown in Figure 3.

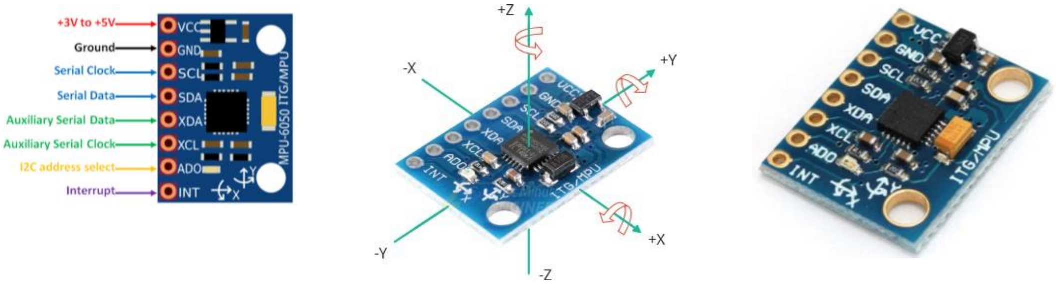
The lithium battery and the charge controller circuit are other components to provide stable current and voltage for the system in the range of 40VDC, 2300mAh/10Cells. Moreover, the charge controller circuit is equipped with a short-circuit protection circuit to protect the battery while charging as shown in Figure 4.



(a) (b)

Figure 3. *Battery protection circuit (a) and communication circuit with the center controller (b).*

To communicate between the central controller and the motor controller as well as the sensors, a communication circuit is designed by using a chip DAC4822. This chip has output analog channels with I2C interface to connect to the accelerometer sensor MPU6050 as shown in Figure. 4



(a) Top view (b) Gyroscope axis (b) PCB board

Figure 4. *Gyroscope sensor board MPU6050.*

To accurately measure the rotation angle as well as the speed of the robot, an accelerometer sensor MCU6050 is used to detect deviations and accelerations. Besides, the accuracy level of measurements is improved thanks to the addition of a programable Kalman filter.

III. MOTION EQUATIONS OF THE TWO-WHEELED ROBOT

Figure 1 (b) shows motion principle of the two-wheeled robot. Motion equations of the two-wheeled robot are based on the Lagrangian equation as follows:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau \tag{1}$$

where L is the Lagrangian, q and d is the generalized coordinate and disturbance vector corresponds to ϕ and θ respectively. H_{ij} is an element of the inertia matrix, b_i are nonlinear terms, and other symbols are given in Table 1.

Table 1. *Explanation of symbols.*

Explanation	Symbol	Unit	Explanation	Symbol	Unit
Inertia of rod	J_ϕ	$kg.m^2$	Angle of wheel	θ	rad
Inertia of wheel	J_θ	$kg.m^2$	Angle of rod	ϕ	rad
Gravitational acceleration	g	m/s^2	Actuation torque	τ	$N.m$
Viscous friction coefficient of rod	D_ϕ	$N.m.s/rad$	Mass of rod	m	kg
Viscous friction coefficient of wheel	D_θ	$N.m.s/rad$	Mass of wheel	M	kg

The transfer function matrix of the robot is described as follows:

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix} \tag{2}$$

where the elements of the motion equations of the two-wheeled robot can be calculated by:

$$H_{11} = J_\theta + (M+m)r^2 + 2mrl\cos\phi + J_\phi + ml^2 \tag{3}$$

$$H_{12} = H_{21} = -J_\theta - (M+m)r^2 - mrl\cos\phi \tag{4}$$

$$H_{22} = J_\theta + (M+m)r^2 \tag{5}$$

$$b_1 = -\dot{\phi}^2 mrl\sin\phi - mgl\sin\phi + D_\phi\dot{\phi} \tag{6}$$

$$b_2 = \dot{\phi}^2 mrl\sin\phi + D_\theta\dot{\theta} \tag{7}$$

The required torque applied to the motor axis of the two-wheeled robot is calculated from the equation below:

$$\tau = (H_{22} - \frac{H_{12}H_{21}}{H_{11}})u - \frac{H_{21}}{H_{11}}b_1 + b_2 \tag{8}$$

Therefore, the linear equations of the nonlinear system of an inverted pendulum model are described in [7] will be re-presented as below:

$$y^{(0)} = \int_0^\phi \frac{H_{11}}{H_{12}} d\phi + \theta \tag{9}$$

$$y^{(1)} = \frac{H_{11}}{H_{12}}\dot{\phi} + \dot{\theta} \tag{10}$$

$$y^{(2)} = \frac{\partial}{\partial\phi}\frac{H_{11}}{H_{12}}\dot{\phi}^2 - \frac{b_1}{H_{12}} \tag{11}$$

$$y^{(3)} \simeq \frac{\partial^2}{\partial\phi^2}\frac{H_{11}}{H_{12}}\dot{\phi}^3 - \frac{\partial}{\partial\phi}\frac{b_1}{H_{12}}\dot{\phi} - 2(\frac{\partial}{\partial\phi}\frac{H_{11}}{H_{12}})\frac{b_1}{H_{11}}\dot{\phi}) \tag{12}$$

For simplicity in programming and analyzing, the expansion with the rank of the derivative of fourth is used, then the input value of the nonlinear controller is given by:

$$u = \frac{v - L_f{}^4 h(x)}{L_g L_f{}^3 h(x)} \tag{13}$$

$$L_g L_f{}^3 h(x) = -3\frac{\partial^2}{\partial\phi^2}\frac{H_{11}}{H_{12}}\frac{H_{12}}{H_{11}}\dot{\phi}^2 + \frac{\partial}{\partial\phi}\frac{b_1}{H_{12}}\frac{H_{12}}{H_{11}} + 2\frac{\partial}{\partial\phi}\frac{H_{11}}{H_{12}}\frac{H_{12}}{H_{11}{}^2}b_1 \tag{14}$$

$$L_f{}^4 h(x) = -\frac{\partial^2}{\partial\phi^2}\frac{b_1}{H_{12}}\dot{\phi}^2 + \frac{\partial^3}{\partial\phi^3}\frac{H_{11}}{H_{12}}\dot{\phi}^4 - 5(\frac{\partial^2}{\partial\phi^2}\frac{H_{11}}{H_{12}})\frac{b_1}{H_{11}}\dot{\phi}^2 - 2\frac{\partial}{\partial\phi}\frac{H_{11}}{H_{12}}\frac{\partial}{\partial\phi}\frac{b_1}{H_{11}}\dot{\phi}^2 + \frac{\partial}{\partial\phi}\frac{b_1}{H_{12}}\frac{b_1}{H_{11}} + 2\frac{\partial}{\partial\phi}\frac{H_{11}}{H_{12}}\left(\frac{b_1}{H_{11}}\right)^2 \tag{15}$$

Thus, the coefficient of the basic feedback loop controller is defined as:

$$v = -\sum_{i=0}^{3}\lambda_i(y^{(i)} - y_{ref}^{(i)}) \tag{16}$$

Moreover, the input value of the nonlinear controller in Equation (13) can be represent as below:

$$u = \frac{\lambda_0\left(y_r^{(0)} - y^{(0)}\right) + \lambda_1\left(y_r^{(1)} - y^{(1)}\right) + \mathrm{W}_{CT}\lambda_2\left(y_r^{(2)} - y^{(2)}\right) + \lambda_3\left(y_r^{(3)} - y^{(3)}\right) + \left(L_f^4 h(x) - L_f^4 h(x)_r\right)}{L_g L_f^3 h(x)} \tag{17}$$

In this research, we focus on scheduling the gain W_{CT} in Equation (17), this coefficient directly affects the robot's acceleration to support the user maintain balancing. The proposed law of the gain-scheduling method is determined by as follows:

if (abs(phi) < 0.01 && abs(phi) > 0.00)
W_{CT} = 5.0;
else if (abs(phi) < 0.02 && abs(phi) > 0.01)
W_{CT} = 6.0;
else if (abs(phi) < 0.03 && abs(phi) > 0.02)
W_{CT} = 7.0;
else if (abs(phi) < 0.04 && abs(phi) > 0.03)
W_{CT} = 8.0;
else if (abs(phi) < 0.05 && abs(phi) > 0.04)
W_{CT} = 9.0;
else if (abs(phi) < 0.06 && abs(phi) > 0.05)
W_{CT} = 15.0;

(18)

By using this law, we can programing on the center controller with a C/C++ langue to control the two-wheeled robot following the behaviors of the users without falling. All of the experimental results have shown in chapter 4 below.

IV. EXPERIMENTAL RESULTS

The proposed gain-scheduling method is tested on the hardware model with a real user as shown in Figure 5. The two-wheeled robot firmly stands by itself without any support from the outside (Fig. 5-a). The real user is supported by the two-wheeled robot to maintain balancing as shown in Fig. 5-b. In this test, the user applied a force to move the two-wheeled robot forward and backward to check the position response of the two-wheeled robot in accordance with the user's behavior.

(a) Self-balancing *(b) Supported user maintain balancing*

Figure 5. *Test the proposed gain-scheduling method on the two-wheeled robot: the self-balancing robot (a) and the robot supports the user to maintain balancing (b).*

Results without using the proposed gain-scheduling method in Figure 6 show that when the user exerts a force on the two-wheeled robot to move forward and backward, the position response of the two-wheeled robot is very slow and values of the gain u of the controller is just enough to keep the robot in the equilibrium. At aroud $0.4s : 0.5s$ the robot can support the user to stable ($\theta = 0.0rad, \phi = 0.0rad$) without moving as a standing mode of the system.

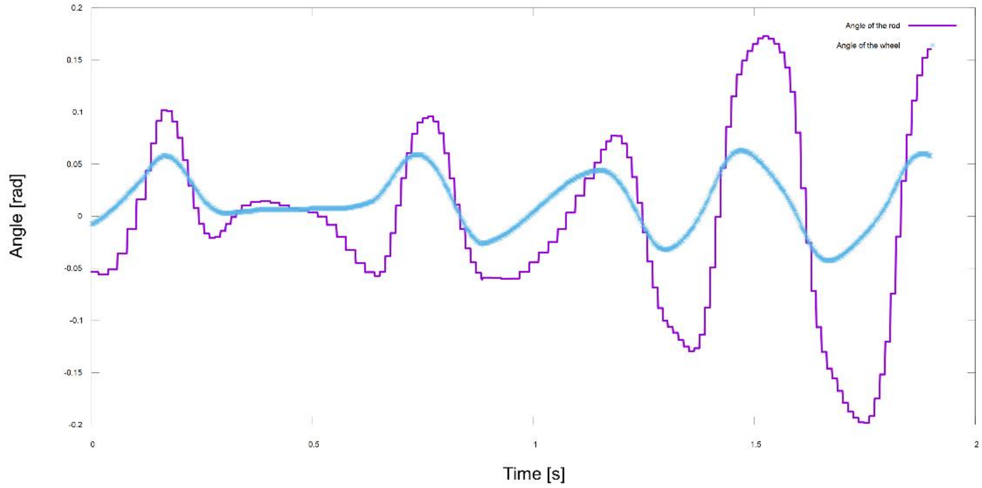


Figure 6. *The angle of the rod (ϕ) and the angle of the wheel (θ) of the two-wheeled robot without using the gain-scheduling method.*

Specifically, in Figure 6 at the $1.5s$ when the robot's ϕ angle decreases from about $0.18rad$ to $0.0rad$, the θ angle of the robot is still at the upper state of $0.08rad$ and starts declining following the deflection angle of the robot. However, the delay between two signals is too big about 0.15 seconds, which proves that the feedback of the robot's state relative to the user's position is not stable to maintain balancing for the user or put the user in danger.

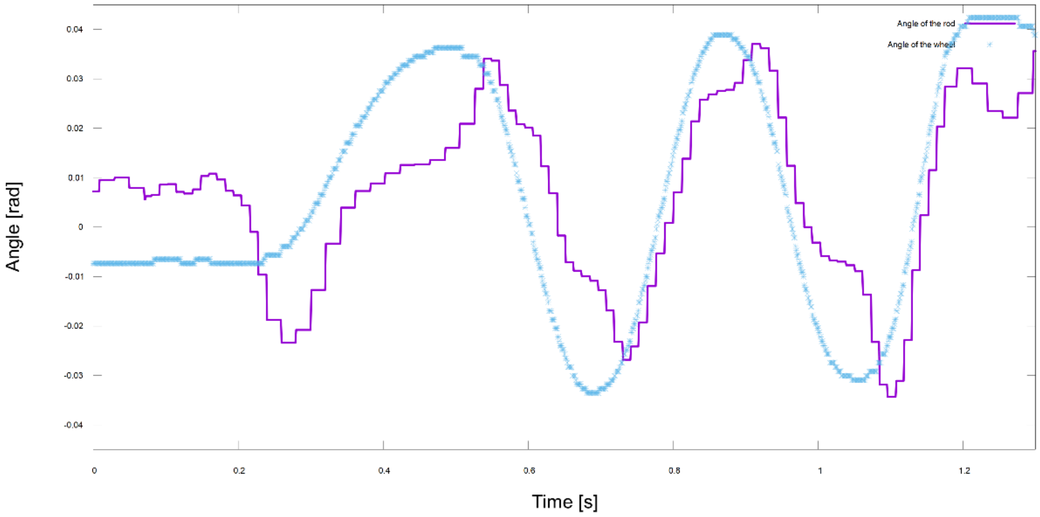
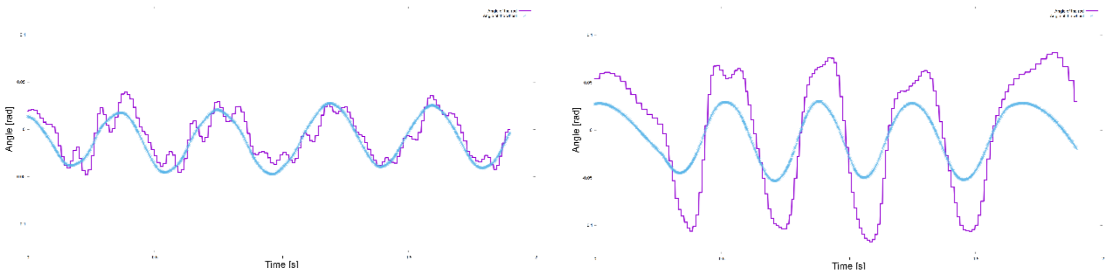


Figure 7. *The angle of the rod (ϕ) and the angle of the wheel (θ) of the two-wheeled robot with the gain-scheduling method.*

Contrary to the results in Figure 6, when the gain-scheduling method is applied to control the two-wheeled robot, the robot's response to the user's behavior is greatly improved. The robot reponses instantaneously to the user's impact as shown in Figure 7. In detail, at the $0.7s$ when the angle of the rod and the angle of the wheel of the two-wheeled robot is at about $-0.025rad$. When the user makes a force on the robot, the deflection angle of the rod increases, and the position of the robot also follows immediately. It proves that the two-wheeled robot's response to the user's impact is instantaneous, suitable for the application of the two-wheeled robot to assist the user in stabilizing the balance while moving.



a) *b)*

Figure 8. *Feedback of the two-wheeled robot in cases with and without the gain-scheduling method.*

Figure 8 shows the deflection angle of the robot in two cases with and without applying the gain-scheduling method on the robot model when the user moves forward and backward around the equilibrium position of the robot. Specifically, the results with the gain-scheduling method in Fig. 8 (a) show that the deflection angle of the robot and the deflection angle of the wheel are similar when the gain-scheduling method is applied. These two values closely follow each other in the entire test period even the deflection angle of the robot is far from the equilibrium point. On the contrary, in the case without using the gain-scheduling method, the results in Fig. 8 (b) reveal that when the deflection angle of the robot body oscillates far from the equilibrium position, the feedback on the deflection angle of the wheel only responds to about 60% compared to the deflection angle of the robot. This leads to the assistance from the robot to the user being unstable and dangerous to the user. Thereby proving that the application of the gain-scheduling method to control the proposed robot is highly effective.

Next, in order to verify the performance of the gain-scheduling method, the proposed two-wheeled robot will be tested with the LQR method [17], which is one of the most popular method to control the robot based on the inverted pendulum. The system matrices are required and are obtained as:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0605 & 96.0703 & -139.2679 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0139 & -33.8276 & -49.0380 \end{bmatrix},\ B=\begin{bmatrix} 0 \\ -36.7358 \\ 0 \\ 4.4353 \end{bmatrix}, C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, D=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

By choosing $R=1$ and $Q=\begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

The gain vector of the LQR is attained as:

$$K=[-62.5910 \quad -1.7543 \quad -3.0959 \quad 2.3568]$$

The experimental results when applied LQR method on the hardware of the two-wheeled robot are shown in Fig. 9.

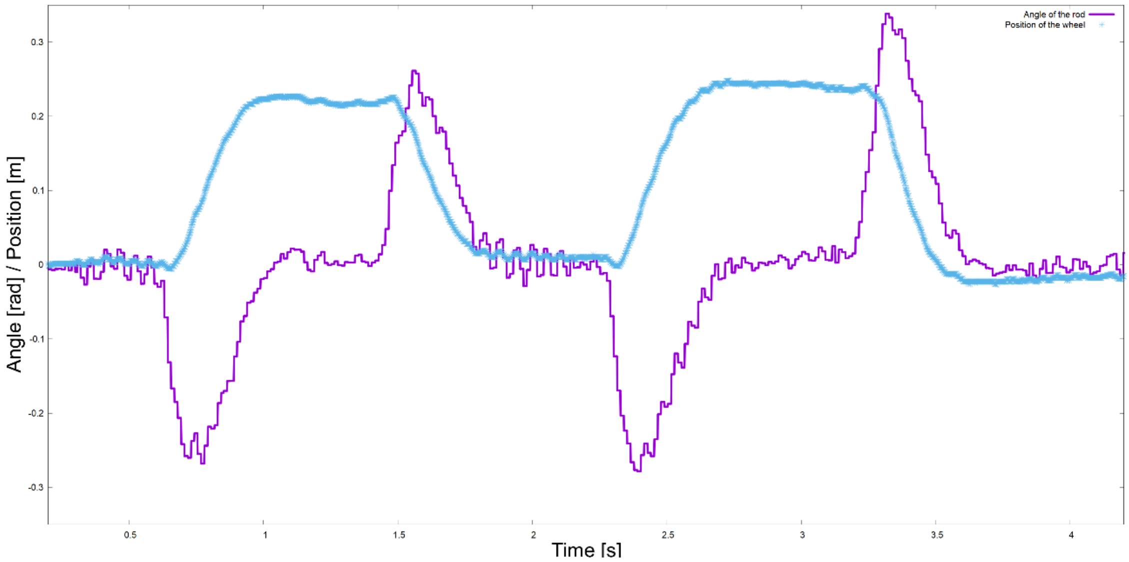


Figure 9. *Feedback of the two-wheeled robot to the user when LQR method is applied.*

Results in Fig. 9 show that the LQR method works quite well and stably with the deflection angle of the inverted pendulum (phi) around the equilibrium point. In the period from 0s to 0.7s, when the user needs to maintain balance, the tilt angle of the robot (phi) oscillates around the equilibrium position (0 rad), so the position of the robot is almost unchanged and always kept in an upright position for both the robot and the user.

Next, when the user tends to move backwards (from 0rad to -0.25rad), the wheel of the robot moves with a very small deflection angle at a time interval of 0.7s. The user then creates a force to make a deflection angle of the handle to move it forward from -0.25rad to 0rad, the robot has a position feedback from 0m to +0.22m to regain the new balance position. This process is repeated at subsequent intervals in the test period.

It can be seen that when the LQR method is used, the inverted pendulum model can work effectively around the equilibrium position. However, when there are external forces from the user, it does not timely respond to these forces. Thus, we can see that when the gain-scheduling method is applied, this problem is completely solved. The two-wheeled robot works completely stable even the deflection angle is big to support the user moving as above-mentioned.

V. CONCLUSION

In this article, a completed hardware and performance test results of the two-wheeled robot to support the elder in walking were presented. The two-wheeled robot is based on the inverted pendulum which is linearized by utilizing the lie algebra method. The gain-scheduling method for a nonlinear system was proposed to apply to a two-wheeled robot. Thus, the performance of the two-wheeled robot is greatly improved. This is verified by experimental results which show that the time response of the two-wheeled robot is instantaneous. As a result, the two-wheeled robot is really effective to support the elder or disable in walking or standing.

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