An Approach to The Analysis and Design Of Fuzzy Control System

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***Abstract*—An approach to the analysis and design of continuous T-S fuzzy control system is proposed in this paper. The fuzzy logic controller has two consequents in each rule. They are numerator part and denominator part and are equivalent to any proper practical controller. It is shown that the overall closed-loop system behaves like an uncertain polytope of polynomials and the system stability can be checked by using some graphical robust stability criteria. The complex process on finding a common Lyapunov function to guarantee the system stability can be omitted. An illustrative example will be given to demonstrate the ability of the design procedure in the proposed approach.**

Keywords*—*T-S fuzzy model; Stability; Fuzzy logic control

I. INTRODUCTION

Among various fuzzy modelling themes [1], the Takagi-Sugeno (T-S) model [2] has been one of the most popular modeling frameworks. T-S fuzzy models can be as universal approximator, then any smooth nonlinear control systems can be approximated by T-S fuzzy models. Different feedback control schemes can be applied to T-S fuzzy models. The most commonly used control law is based on the so-called parallel distributed compensation (PDC) concept [3-5] for which the fuzzy controller shares the same fuzzy rules and sets as the T-S fuzzy model. According to the principle of PDC, a linear controller is designed for each local linear plant to ensure stability and desired performance of the local linear closed loop system using methods from the linear control theory, thus compensating a corresponding conclusion in the rules of T-S plant model. The final nonlinear control is a fuzzy blending of the individual rules control actions. A sufficient condition to ensure the stability of the overall system is obtained by finding a common Lyapunov function which can satisfy all fuzzy sub-systems. The main method to find the common Lyapunov function is solving the linear matrix inequalities (LMIs) by using numerical technique [6]. The main drawbacks of PDC design approach are the difficulty in finding the common Lyapunov function for the large number of fuzzy subsystems and the complex calculations without guaranteeing a solution. To overcome these weaknesses, a new fuzzy logic controller is proposed in [7], which has two consequents in each rule: a numerator part and a denominator part. Besides, the coefficients of both numerator and denominator are computed such that the overall closed-loop systems be haves like linear system. There are some limitations in applying the above proposed concept for continuous time systems. The first is that the feedback controller is not proper, it leads to PD-type or PID compensators, which have infinite bandwidth, whereas real components and compensators always have finite bandwidth. The second reason is that the perfect desired closed-loop system can be obtained if and only if all order derivatives of the system output (or all the states) are sensed and feed them back. In reality, only the system output and some certain its derivatives are measurable. Consequently, any practical compensator must rely only on system outputs, inputs and a few their derivatives.

In this paper, a new method of T-S fuzzy continuous-time system analysis and design is proposed. This is extension of the concept in [7] with some modifications to overcome the above limitations.

II. MODIFIED TAKAGI-SUGENO FUZZY MODEL AND SYSTEM STABILITY ANALYSIS

The fuzzy model is described by differential equation, not by state equation as in the common T-S model. The ith fuzzy IF-THEN rule for describing nth order plant is of the following form:

Plant model Rule R_i:
IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i
THEN $y^{(n)}(t) = -a_{n-1}^i y^{(n-1)}(t) - \cdots - a_0^i y(t) + b_m^i u^{(m)}(t) + \cdots + b_0^i u(t)$ (1)

where R_i denotes the *i*th fuzzy inference rule, *r* is the number of inference rules, $m \le n$, M_j^i, with $i \in \Omega_i$ and $j \in \Omega_p$, are the fuzzy sets, u and y are input and output of the plant, respectively, $a_{n-1}^i, \ldots, a_0^i$ and $b_m^i, b_{m-1}^i, \ldots, b_0^i$ are coefficients of the linear differential equation. The vector of premise variables is defined as $z(t) = [z_1(t) \ldots z_p(t)]$.

Using the center-of-gravity method for defuzzification, the T-S fuzzy model (1) can be represented in the following compact form:

$$y^{(n)}(t) = -\sum_{i=1}^{r} h_i(z)\, a_{n-1}^i y^{(n-1)}(t) - \cdots - \sum_{i=1}^{r} h_i(z)\, a_0^i y(t) + \sum_{i=1}^{r} h_i(z)\, b_m^i u^{(m)}(t) + \cdots + \sum_{i=1}^{r} h_i(z)\, b_0^i u(t) \quad (2)$$

where the normalized membership function $h_i(z)$ is defined as:

$$h_i(z)=\frac{\omega_i(z)}{\sum_{i=1}^{r}\omega_i(z)},\omega_i(z)=\prod_{j=1}^{p}\mu_j^i(z_j),i\in\Omega_r \quad (3)$$

The grades of membership of the premise variables in the respective fuzzy set M_j^i are given as $\mu_j^i(z_j)$. Note that the normalized membership functions satisfy the following convex sum property:

$$0\le h_i(z)\le 1,\sum_{i=1}^{r}h_i(z)=1 \quad (4)$$

According to the premise of each rule of the T-S fuzzy plant model, the PDC design technique derives a control rule under the same premise. However, there are two consequents in the consequent part of each control rule of the FLC: a numerator part and a denominator part of the control signal [7]. Assume that the reference signal $r(t)=r$ is a constant and the error $e(t)=r(t)-y(t)$ is an input to the controller.

Control Rule R_i:

IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i

THEN $num\,u(t)=-b_m^i u^{(m)}(t)-\cdots-b_1^i u'(t)-c_{m_1}^i y^{(m_1)}(t)\ldots-c_0^i y(t)+c_0^i r$

$den\,u(t)=b_0^i,$ (5)

where $num\,u(t)$ is the numerator of $u(t)$, $den\,u(t)$ is the denominator of $u(t)$.

Remark. The proposed local control rule

$$u(t)=\frac{-b_m^i u^{(m)}(t)-\cdots-b_1^i u'(t)-c_{m_1}^i y^{(m_1)}(t)\ldots-c_0^i y(t)+c_0^i r}{b_0^i}$$

is equivalent to realization of the following controller transfer function

$$C_i(s)=\frac{U(s)}{E(s)}=\frac{c_{m_1}^i s^{m_1}+c_{m_1-1}^i s^{m_1-1}+\cdots+c_0^i}{b_m^i s^m+b_{m-1}^i s^{m-1}+\cdots+b_0^i}$$

where $e(t)=r(t)-y(t)$.

Following the control rule in [7], m_1 must be equal to n so then the controller transfer function is not proper. It is required here only that $m_1\le m$.

The inferred output of the proposed FLC is

$$u(t)=\frac{num[u(t)]}{den[u(t)]}=\frac{-\sum_{i=1}^{r}h_i(z)b_m^i u^{(m)}(t)-\ldots-\sum_{i=1}^{r}h_i(z)b_1^i u'(t)}{\sum_{i=1}^{r}h_i(z)b_0^i}+\frac{-\sum_{i=1}^{r}h_i(z)c_{m_1}^i y^{(m_1)}(t)+\cdots-\sum_{i=1}^{r}h_i(z)c_0^i y(t)+\sum_{i=1}^{r}h_i(z)c_0^i r}{\sum_{i=1}^{r}h_i(z)b_0^i} \quad (6)$$

Substituting this control signal into the T-S fuzzy plant model of (2), the closed-loop fuzzy control system becomes:

$$\begin{aligned}
y^{(n)}(t)=&-\sum_{i=1}^{r}h_i(z)a_{n-1}^i y^{(n-1)}(t)-\cdots\\
&-\sum_{i=1}^{r}h_i(z)a_0^i y(t)+\sum_{i=1}^{r}h_i(z)b_m^i u^{(m)}(t)\\
&+\cdots+\sum_{i=1}^{r}h_i(z)b_0^i u(t)\\
=&-\sum_{i=1}^{r}h_i(z)a_{n-1}^i y^{(n-1)}(t)-\cdots\\
&-\sum_{i=1}^{r}h_i(z)a_0^i y(t)+\sum_{i=1}^{r}h_i(z)b_m^i u^{(m)}(t)\\
&+\cdots+\sum_{i=1}^{r}h_i(z)b_1^i u'(t)\\
&-\sum_{i=1}^{r}h_i(z)b_m^i u^{(m)}(t)-\cdots\\
&-\sum_{i=1}^{r}h_i(z)b_0^i u'(t)\\
&-\sum_{i=1}^{r}h_i(z)c_{m_1}^i y^{(m_1)}(t)+\cdots\\
&-\sum_{i=1}^{r}h_i(z)c_0^i y(t)+\sum_{i=1}^{r}h_i(z)c_0^i r\\
=&-\sum_{i=1}^{r}h_i(z)a_{n-1}^i y^{(n-1)}(t)-\cdots\\
&-\sum_{i=1}^{r}h_i(z)\left(a_{m_1}^i+c_{m_1}^i\right)y^{(m_1)}(t)-\cdots\\
&-\sum_{i=1}^{r}h_i(z)\left(a_0^i+c_0^i\right)y(t)+\sum_{i=1}^{r}h_i(z)c_0^i r
\end{aligned} \quad (7)$$

The closed-loop system characteristic equation is of the following form:

$$s^n+\sum_{i=1}^{r}h_i(z)a_{n-1}^i s^{n-1}+\cdots+\sum_{i=1}^{r}h_i(z)\left(a_{m_1}^i+c_{m_1}^i\right)s^{m_1}+\cdots+\sum_{i=1}^{r}h_i(z)\left(a_0^i+c_0^i\right)=0$$

$$0\le h_i(z)\le 1,i=1,\ldots,r,\sum_{i=1}^{r}h_i(z)=1 \quad (8)$$

The left side of the equation (8) is the characteristic polynomial and in fact, it is the polytope of polynomials:

$$H(s) = s^n + \sum_{i=1}^{r} h_i(z) a_{n-1}^i s^{n-1} + \cdots + \sum_{i=1}^{r} h_i(z)\left(a_{m_1}^i + c_{m_1}^i\right) s^{m_1} + \cdots + \sum_{i=1}^{r} h_i(z)\left(a_0^i + c_0^i\right) = \sum_{i=1}^{r} h_i(z) H_i(z) \quad (9)$$

$$0 \le h_i(z) \le 1, i = 1, \dots, r, \sum_{i=1}^{r} h_i(z) = 1$$

where

$$H_i(s) = s^n + a_{n-1}^i s^{n-1} + \cdots + \left(a_{m_1}^i + c_{m_1}^i\right) s^{m_1} + \cdots + \left(a_0^i + c_0^i\right) \quad (10)$$

That is different in compared with a certain characteristic linear polynomial in [7].

Therefore, the system stability analysis can be done by using the robust stability criteria derived in [8-9], what requires to check $\frac{r(r-1)}{2}$ polynomial segments to Hurwitz and the testing stability of an edge $\lambda H_i(s) + (1-\lambda) H_j(s)$ can be made by Nyquist test: the plot $z(j\omega) = \frac{H_i(j\omega)}{H_j(j\omega)}$ should not intersect negative real semi axis.

Theorem 1: The fuzzy control system (1), (5) is stable if and only if

- The polynomials $H_i(s), i = 1, \dots, r$ are stable,
- All $\frac{r(r-1)}{2}$ plot $z_{ij}(j\omega) = \frac{H_i(j\omega)}{H_j(j\omega)}, i, j = 1, \dots, r, i < j$ do not intersect negative real semi axis.

III. ROBUST FUZZY PDC CONTROL OF NONLINEAR PLANT

Suppose that by carrying the experiments or the physical analysis of the plant, the local models in different operation points are obtained in form (1) with $m < n$. For each local plant model, the general optimal local controller is derived as follows:

Control rule R_i: IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i

THEN $num\, u(t) = -u^{(n_1)}(t) - d_{n_1-1}^i u^{(n_1-1)}(t) - \cdots - d_{m_1}^i u'(t) - c_{m_1}^i y^{(m_1)}(t) - \cdots - c_0^i y(t) + c_0^i r$

$den\, u(t) = d_0^i$, (11)

where $m_1 < n_1$.

Remark. In deed, the above rule consequent is equivalent to the following controller transfer function

$$C_i(s) = \frac{U(s)}{E(s)} = \frac{c_{m_1}^i s^{m_1} + \cdots + c_0^i}{s^{n_1} + d_{n_1-1}^i s^{n_1-1} + \cdots + d_0^i} \quad (12)$$

The problem is to check the stability of the overall closed-loop system (1), (11).

Note that the consequent of the local plant model rule (1) describes the local transfer function

$$P_i(s) = \frac{Y(s)}{U(s)} = \frac{b_m^i s^m + b_{m-1}^i s^{m-1} + \cdots + b_0^i}{s^m + a_{n-1}^i s^{n-1} + \cdots + a_0^i}$$

and they are equivalent to the following augmented transfer function

$$P_i^*(s) = \frac{Y(s)}{U(s)} = \frac{b_m^i s^m + b_{m-1}^i s^{m-1} + \cdots + b_0^i}{s^m + a_{n-1}^i s^{n-1} + \cdots + a_0^i} \cdot \frac{s^{n_1} + d_{n_1-1}^i s^{n_1-1} + \cdots + d_0^i}{s^{n_1} + d_{n_1-1}^i s^{n_1-1} + \cdots + d_0^i} \quad (13)$$

Similarly, the local controller transfer function can be augmented as follows:

$$C_i^*(s) = \frac{U(s)}{E(s)} = \frac{c_{m_1}^i s^{m_1} + \cdots + c_0^i}{s^{n_1} + d_{n_1-1}^i s^{n_1-1} + \cdots + d_0^i} \cdot \frac{b_m^i s^m + \cdots + b_0^i}{b_m^i s^m + \cdots + b_0^i}$$

Therefore, the following results can be obtained.

Lemma 1. The local plant model (1) is equivalent to the following model

Plant model Rule R_i: IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i

THEN

$$y^{(n+n_1)}(t) = -(d_{n_1-1}^i + a_{n-1}^i) y^{(n+n_1-1)}(t) - \cdots - \left(a_1^i d_0^i + a_0^i d_1^i\right) y'(t) - a_0^i d_0^i y(t) + b_m^i u^{(m+n_1)}(t) + \cdots + (b_1^i d_0^i + b_0^i d_1^i) u'(t) + b_0^i d_0^i u(t) \quad (14)$$

Lemma 2. The local controller model (11) is equivalent to the following model

Control Rule R_i: IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i

THEN $num\, u(t) = -b_m^i u^{(m+n_1)}(t) - \cdots - (b_1^i d_0^i + b_0^i d_1^i) u'(t) - c_{m_1}^i b_m^i y^{(m_1+m)}(t) - \cdots - \left(b_1^i c_0^i + b_0^i c_1^i\right) y'(t) - b_0^i c_0^i y(t) + b_0^i c_0^i r$

$den\, u(t) = b_0^i d_0^i$, (15)

It is not difficult to derive the dynamical equation of the fuzzy closed-loop systems (14), (15) as follows:

$$y^{(n+n_1)}(t) = -\sum_{i=1}^{r} h_i(z)\left(d_{n_1-1}^i + a_{n-1}^i\right) y^{(n+n_1-1)}(t) - \cdots - \sum_{i=1}^{r} h_i(z)\left(a_{m+m_1}^i + c_{m_1}^i b_m^i\right) y^{(m+m_1)}(t) - \sum_{i=1}^{r} h_i(z)\left(a_0^i d_0^i + b_0^i c_0^i\right) y(t) + \sum_{i=1}^{r} h_i(z) b_0^i c_0^i r \quad (16)$$

The system characteristic polynomial becomes

$$H(s) = s^{n+n_1} + \sum_{i=1}^{r} h_i(z)\left(d_{n_1-1}^i + a_{n-1}^i\right)s^{n+n_1-1} + \cdots + \sum_{i=1}^{r} h_i(z)\left(a_0^i d_0^i + b_0^i c_0^i\right) \quad (17)$$

Denote

$$H_i(s) = s^{n+n_1} + \left(d_{n_1-1}^i + a_{n-1}^i\right)s^{n+n_1-1} + \cdots + \left(a_0^i d_0^i + b_0^i c_0^i\right), i = 1, \dots, r \quad (18)$$

Theorem 2. The fuzzy closed-loop system (1), (11) is stable if and only if the polytope of polynomials

$$H(s) = \sum_{i=1}^{r} h_i(z)H_i(z), 0 \le h_i(z) \le 1, \sum_{i=1}^{r} h_i(z) = 1 \quad (19)$$

is robust stable.

The robust stability of the family (19) can be done by applying the Theorem 1.

IV. EXAMPLE

Consider the example of PDC-based FLC for temperature in [10]. The plant is a laboratory dryer. The air temperature y is controlled by changing the voltage u to a Pulse-Width Modulator (PWM) thus varying the duty ratio of switching of an electrical heater and a fan. It is able to distinguish three overlapping linearization zones: Zone (1) for $y = 20 \div 50^o C$, Zone (2) for $y = 40 \div 57^o C$, Zone (3) for $y = 50 \div 80^o C$. In each zone a plant model.

$$P_i(s) = \frac{K^i}{(T_1^i(s)+1)(T_2^i(s)+1)(s+1)}, i = 1,2,3$$

is obtained by carrying experimental and optimized using GA algorithm.

The optimal PI controllers of the following form

$$C_i(s) = k_P^i + \frac{k_I^i}{s} = \frac{k_P^i s + k_I^i}{s}, i = 1,2,3$$

are designed for each local plant model in each zone. The plant model parameters and controller parameters are shown in Tab. 1 [7].

TABLE I. THE SYSTEM PARAMETERS

i	K^i	T_1^i	T_2^i	k_P^i	k_I^i
1	13.71	57.3	11.6	0.27	0.0027
2	12.2	96.5	15.4	0.36	0.003
3	8.3	49.1	2.5	0.83	0.00415

To check the stability of the designed global nonlinear system, it is necessary to turn the plant model and PI controller description into a state space representation and solve six LMIs. This complexity can be reduced using Theorem 2. It is clear that the examining example can be considered as the T-S fuzzy system (1), (11) where: $m = 0, n = 3, m_1 = n_1 = 1$, and

$$a_2^i = \frac{T_1^i T_2^i + T_1^i + T_2^i}{T_1^i T_2^i}, a_1^i = \frac{T_1^i + T_2^i + 1}{T_1^i T_2^i}, a_0^i = \frac{1}{T_1^i T_2^i}, b_0^i = \frac{K^i}{T_1^i T_2^i}, c_0^i = k_I^i, c_1^i = k_P^i, d_0^i = 0, d_1^i = 1$$

Following (19), the family of system characteristic polynomials has the following form:

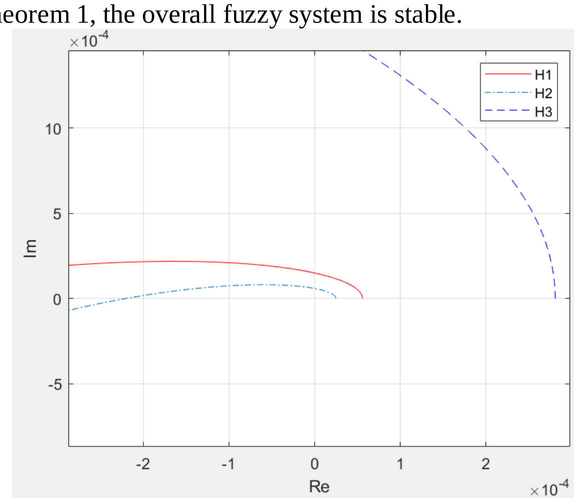
$$H(s) = \sum_{i=1}^{3} h_i(z)H_i(z)$$

$$H_i(s) = s^4 + a_2^i s^3 + a_1^i s^2 + (a_2^i + k_P^i b_0^i)s$$

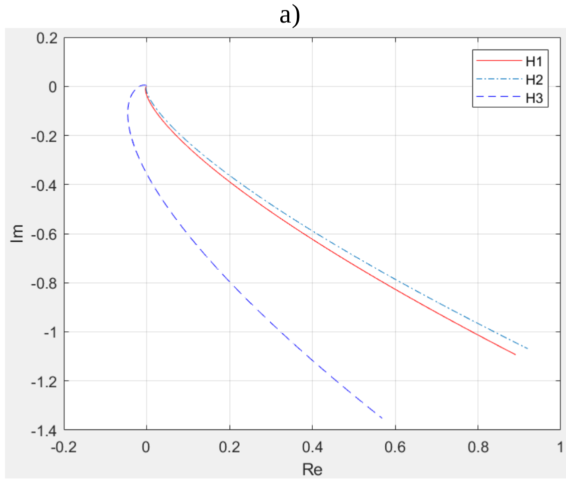
where

$$0 \le h_i(z) \le 1, i = 1, \dots, r, \sum_{i=1}^{r} h_i(z) = 1$$

The Mikhailop plots of the polynomials $H_i(s), i = 1,2,3$ are shown in Fig. 1 and the Nyquist plots $z_{12}(j\omega), z_{13}(j\omega), z_{23}(j\omega)$ are shown in Fig.2. According to Theorem 1, the overall fuzzy system is stable.



a)



b)

Figure 1. The Mikhailop plots: a) starts on the positive real semi-axis, does not hit the origin; b) successively generates an anti-clockwise motion through n quadrants.

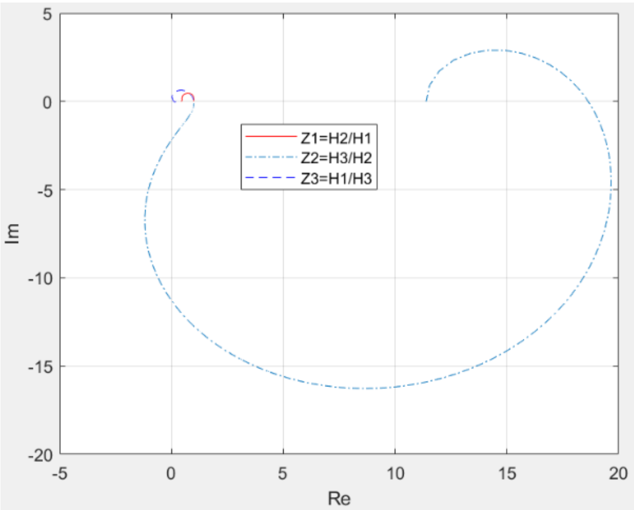


Figure 2. The plots $z_{12}(j\omega), z_{13}(j\omega), z_{23}(j\omega)$

V. CONCLUSION

This paper proposes a new method for the stability analysis and design of continuous T-S fuzzy model-based systems. The proposed method is the extension of the derived concept in [7], which is suitable only for discontinuous systems, where it is not required the proper characteristic of controller. Based on the new proposed method, the closed-loop T-S fuzzy control system behaves like a polytope of linear systems and the system stability can be easily checked by using some graphical robust stability criteria, without applying a complex process to find a common Lyapunov function as in the existing approaches. The number of plots that is $\frac{r(r-1)}{2}$ in comparing with $\frac{r(r+1)}{2}$ LMIs needs to be solved. The proposed design method is applied to design FLCs for a temperature control system, where the PI controllers are used for local systems. The overall system stability is then checked by proposed criterium in the paper.

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